On the determination of the pion effective mass in nuclei from pionic atoms

E. Friedman and A. Gal
Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel

Abstract

The binding energies of the deeply bound 1s and 2p states in pionic atoms of ^{207}Pb, recently established experimentally in the ^{208}Pb(d,^{3}He) reaction, have been used by several groups to derive the pion effective mass in nuclear matter. We show that these binding energies are fully consistent with 'normal' pionic atoms and that the real part of the pion-nucleus potential at the center of ^{207}Pb is 28±3 MeV and not 20 MeV as suggested previously.

PACS: 36.10.Gv; 14.40.Aq
Keywords: pionic atoms; deeply bound pionic states; pion-nucleus potential; pion effective mass

Corresponding author: E. Friedman,
Tel: +972 2 658 4667, Fax: +972 2 658 6347,
E mail: elifried@vms.huji.ac.il

February 9, 2008

Information on the strong interaction at zero energy between a negatively charged hadron and a nucleus may be obtained from the observation of level shifts and widths in hadronic atoms. Such levels are populated via an atomic cascade process of the hadron where the experimentally observed X-ray spectra are terminated at a level for which the radiation yield becomes smaller than the nuclear absorption. Only for light pionic atoms is the 1s level observed, otherwise the spectrum terminates at the 2p, 3d or higher levels. Simple extrapolations led to the expectation that 1s and 2p states in heavy pionic atoms would be quite broad due to the nuclear absorption. The first to show that 1s and 2p states in heavy pionic atoms are so narrow as to make them well defined, were Friedman and Soff [1]. They calculated shifts and widths for pionic atom states well beyond the experimentally reachable region and showed that the atomic wavefunctions of these deeply bound states are pushed out of the nucleus by the repulsive s-wave part of the potential such that their overlap with the nucleus and with the imaginary part of the potential becomes very small. Similar conclusions about the small widths expected for deeply bound pionic atom states were reached by Toki and Yamazaki [2,3], who also considered methods, other than radiative processes, to populate such states. The pionic 1s and 2p states in ^{207}Pb were observed recently by Yamazaki et al. [4,5] in the ^{208}Pb(d,^{3}He) reaction, yielding for the binding energies the values [5]

$$\mathrm{B_{1s}} = 7.1 \pm 0.2 \text{ MeV} \qquad \mathrm{B_{2p}} = 5.31 \pm 0.09 \text{ MeV}. \tag{1}$$

The availability of such results raises the question of the consistency of these deeply bound states with the 'normal' pionic atom states, within the commonly accepted pion-nucleus interaction model. It also focuses attention [6] on the pion effective mass in the nuclear medium. These two points are the topics discussed in this Letter.

The pion-nucleus potential at zero energy is traditionally written [7] in the form

$$2\mu V_{opt}(r) = q(r) + \vec{\nabla} \cdot \alpha(r) \vec{\nabla} \tag{2}$$

with the s-wave part given by

$$\begin{aligned} q(r) = &-4\pi(1 + \frac{\mu}{M})\{b_0[\rho_n(r) + \rho_p(r)] + b_1[\rho_n(r) - \rho_p(r)]\} \\ &-4\pi(1 + \frac{\mu}{2M})4B_0\rho_n(r)\rho_p(r). \end{aligned} \tag{3}$$

In this expression ρ_n and ρ_p are the neutron and proton density distributions normalized to the number of neutrons N and number of protons Z, respectively, μ is the pion-nucleus reduced mass and M is the mass of the nucleon. The real coefficients b_0 and b_1, according to the low density limit [8], are expressed by the π^-p elastic scattering and charge exchange scattering lengths, which have been determined recently from pionic hydrogen [9]:

$$b_0 = -0.0077 \pm 0.0072 \ m_\pi^{-1}, \qquad b_1 = -0.0962 \pm 0.0071 \ m_\pi^{-1}. \tag{4}$$

These values, as summarized in Ref. [6], agree very well with those calculated by chiral perturbation theory. The parameter B_0 is obtained phenomenologically from fits to pionic atom data and its imaginary part represents s-wave absorption on two nucleons, which is dominated by absorption on a neutron-proton pair. A real part for B_0 cannot be excluded and indeed is found to be required by fits to the data, even when b_0 and b_1 are treated as

free parameters. This real part is referred to as the 'missing' s-wave repulsion because it turns out to be repulsive and substantially larger than the imaginary part (Ref. [10] and references therein), contrary to expectations. A correlation between the parameters b_0 and ReB_0 was noted long ago [11] on the basis of analyses of older and much more restricted data sets, suggesting that b_0 and ReB_0 can be lumped together. However, recent analyses of considerably more extended data bases [10,12] find each of the parameters to be reasonably well determined (see table 3 of Ref. [10]). Finally, when discussing the real part of the s-wave potential one has to realize that the isoscalar coefficient b_0 is exceptionally small. Therefore an explicit second order term is often included in the isoscalar part, with b_0 replaced by

$$\overline{b}_0 = b_0 - \frac{3}{2\pi}(b_0^2 + 2b_1^2)k_F \tag{5}$$

where k_F is the Fermi momentum taken either as a constant, or calculated for the local nuclear density. In the present work we adopt this additional term with the latter prescription.

The data base for normal pionic atoms used in the present work contains 54 data points for 1s to 4f states covering the range from oxygen to uranium. It was shown in [10] that it leads to essentially the same results as the very extended data base of Konijn et al. [12] which contains 140 points. As a starting point we note that an unconstrained fit to the data for 'normal' states leads to χ^2/N, the χ^2 per point, of 2.0, a value which is used as a reference to the quality of subsequent fits. The calculated binding energies for the 1s and 2p states in ^{207}Pb are 6.77 and 5.10 MeV, respectively, compared to the experimental values Eq.(1), which means a somewhat inferior agreement between calculation and experiment for the deeply bound states compared to the normal states. The value of the s-wave potential at the center of the ^{207}Pb nucleus, V_S, is found to be 29 MeV (repulsive). Next we adopted a more constrained approach where the values of b_0 and b_1, and of the linear terms in the p-wave $\alpha(r)$ part of the potential (not discussed in the present work), were held fixed at the corresponding free πN values in order to respect the low density limit. This fit leads to χ^2 per point of 2.9 and the predicted values of the 1s and 2p binding energies are then 6.84 and 5.14 MeV, respectively. The χ^2 per point for these two states is now 2.6, thus suggesting full consistency with the normal states. The s-wave potential at the center of ^{207}Pb is now V_S=27.0 MeV, with ReB_0=-0.062 ± 0.006 m$_\pi^{-4}$, ImB_0=0.056 ± 0.003 m$_\pi^{-4}$, thus demonstrating the importance of the ReB_0 term in addition to the free πN s-wave interaction terms, a point which is at variance with conclusions of Ref. [6]. Furthermore, the resulting value for V_S disagrees with that found by considering *only* the deeply bound states [5,6] without checking for consistency with the normal data. Indeed by setting ReB_0=0 and fitting b_0 and b_1 to the experimental 1s and 2p binding energies (and using the standard p-wave potential [10]), we find V_S=16.9 MeV in agreement with [5,6]. However, this potential results in χ^2/N=54 for the normal states, which is totally unacceptable. We therefore proceed with a more systematic study of the connection between ReB_0 and V_S.

The role of the parameter ReB_0 can be assessed by gridding on its value while performing fits to the data, varying the other parameters, *including* b_0 *and* b_1. Figure 1 shows the best fit values of b_0 and b_1 together with their uncertainties, along with the fixed values of ReB_0 listed on the right hand side. Also shown as shaded bands are the experimental values [9] of b_0 and b_1 as obtained from pionic hydrogen. The correlation between these two parameters for the free pion-nucleon interaction is determined to very high precision [9], as is depicted by the two dashed lines within the central shaded area. Figure 2 shows the corresponding

values of χ^2/N, of V_S and of the calculated binding energies of the 1s and 2p states in ^{207}Pb. The experimental values of these two binding energies are shown as shaded areas. The two horizontal error bars show the best χ^2/N and the corresponding value of V_S for the unconstrained fit mentioned above. The dots in this figure correspond to the dots in Fig. 1. The calculated binding energies for the 1s and 2p states hardly vary, demonstarting the b_0-ReB_0 ambiguity noted by Toki et al. [3]. The differences between calculated and experimental results for the 1s and 2p binding energies show a systematic trend. However, this difference amounts to χ^2/N=2.5 for these two states, which is about the same as obtained for the 54 data points of the normal pionic atom states. It is therefore concluded that the two new results for the deeply bound pionic states [5] are fully consistent with the normal states and do not seem to convey any new information about the pion nucleus interaction.

The values of V_S, which are taken as representative of the real s-wave potential in nuclear matter, deserve some discussion. It is clear from Fig. 2 that fits to pionic atom data do not determine V_S at all and its value depends sensitively on the assumptions made about ReB_0. A similar conclusion is obtained when fits are made to the 1s and 2p states only. Nevertheless, it can be shown by performing a 'notch test' [10] that once the coefficients in V_{opt} (Eq.(2)) are fixed, the values of the potential well inside the nucleus are determined within a few percent. If, for example, one assumes that ReB_0=0 then V_S becomes close to 24 MeV. However, this would lead to a deterioration in the fits (Fig. 2) to normal pionic atom data. Imposing the free values on b_0 and b_1 whilst keeping ReB_0=0, reduces further the value of V_S to 17 MeV while causing χ^2/N to become 19, which is unacceptably large in view of the smaller values shown in Fig. 2. Requiring χ^2/N to be around 3 for normal pionic atom states and requiring values for b_0 and b_1 which are removed from the free πN values by no more than one standard deviation, the two figures constrain ReB_0 to be between -0.02 and -0.08 m$_\pi^{-4}$. This leads to the value of the real part of the repulsive s-wave potential inside ^{207}Pb to be 28$\pm$3 MeV. It implies a value of $m_\pi(\rho)$ =170.4$\pm$3.6 MeV, where the pion effective mass at density ρ is given by

$$m_\pi^2(\rho) = m_\pi^2 + 2m_\pi(\rho)V_S(\rho), \qquad m_\pi(0) = m_\pi. \tag{6}$$

For symmetric nuclear matter (ρ_0=0.17 fm^{-3}) where b_1 is ineffective, one then has $m_\pi(\rho_0)$ =167 MeV.

A possible point of concern is the poorly known density distributions for neutrons which enter the optical potentials. For $N = Z$ nuclei it is obvious to use the same distribution for the neutrons as for the protons. For $N > Z$ nuclei we followed the general procedure outlined in [10] using for neutrons distributions with slightly larger rms radii than for the protons. For ^{207}Pb the rms radius for the neutrons was chosen as 0.19 fm larger than the corresponding value for the protons, as suggested by averaging the various results for ^{208}Pb summarised in [13]. However, recent calculations based on relativistic mean field theory [14] suggest a value of 0.26 fm for the difference. The sensitivities of the various calculated values for ^{207}Pb for a change of 0.1 fm in that difference are -95 keV for B$_{1s}$, -55 keV for B$_{2p}$ and -2.7 MeV for V_S. The value of $m_\pi(\rho_0)$, the effective mass in symmetric nuclear matter, depends only very marginally on the assumed radii for the neutron distributions.

A comment on the widths of the deeply bound states is in order. Experimentally [4,5] only an upper limit of 0.8 MeV could be placed on the width of the 2p state. All our potentials that produce acceptable fits to normal pionic atoms predict a width of 0.31-0.33

MeV for the 2p state, and a width of 0.45-0.50 MeV for the 1s state. However, if one sets ReB_0=0 and imposing on b_0 and b_1 the free πN value, then the widths of the 2p and 1s states become 0.44 and 0.8 MeV, respectively. Much improved experimental accuracies will be required in the determination of widths of deeply bound states if these are to be useful as a source of information on the π-nucleus interaction.

In conclusion, fits to normal pionic atom data show that a real part of the two-nucleon absorption term of the s-wave part of the potential is required and is well determined by the data, once the low density limit is imposed on the s-wave part of V_{opt} (Eq. (2)), thus demonstrating that the problem of 'missing' s-wave repulsion persists [10]. Imposing the low density limit on the s-wave part of the potential, its value in the interior of the ^{207}Pb nucleus is found to be 28$\pm$3 MeV, leading to a pion effective mass in *symmetric* nuclear matter of 167$\pm$3.5 MeV. It is also shown that any *good* fit to normal pionic atom data, which approximately respects the low density limit, leads to calculated binding energies for the deeply bound 1s and 2p states in ^{207}Pb that agree with experiment at the same level as for normal states and to V_S in the range specified above. One might suspect that the same mechanism which causes the deeply bound states to be narrow also masks the deep interior of nuclei where new effects could possibly be observed.

We wish to thank C.J. Batty for useful comments. This research was partially supported by the Israel Science Foundation.

REFERENCES

[1] E. Friedman and G. Soff, J. Phys. G: Nucl. Phys. 11 (1985) L37.
[2] H. Toki and T. Yamazaki, Phys. Lett. B 213 (1988) 129.
[3] H. Toki, S. Hirenzaki, T. Yamazaki and R.S. Hayano, Nucl. Phys. A 501 (1989) 653.
[4] T. Yamazaki et al., Z. Phys. A 355 (1996) 219.
[5] T. Yamazaki et al., Phys. Lett. B 418 (1998) 246.
[6] T.Waas, R. Brockmann and W. Weise, Phys. Lett. B 405 (1997) 215.
[7] M. Ericson and T.E.O. Ericson, Ann. Phys. [NY] 36 (1966) 323.
[8] C.B. Dover, J. Hüfner and R.H. Lemmer, Ann. Phys. [NY] 66 (1971) 248.
[9] D. Sigg et al., Nucl. Phys. A 609 (1996) 269; Erratum Nucl. Phys. A 617 (1997) 526.
[10] C.J. Batty, E. Friedman and A. Gal, Phys. Reports 287 (1997) 385.
[11] R. Seki and K. Masutani, Phys. Rev. C 27 (1983) 2799.
[12] J. Konijn, C.T.A.M. de Laat, A. Taal and J.H. Koch, Nucl. Phys. A 519 (1990) 773.
[13] C.J. Batty, E. Friedman, H.J. Gils and H. Rebel, Adv. Nucl. Phys. 19 (1989) 1.
[14] M. Warda, B. Nerlo-Pomorska and K. Pomorski, Nucl. Phys. A (1998) in press.

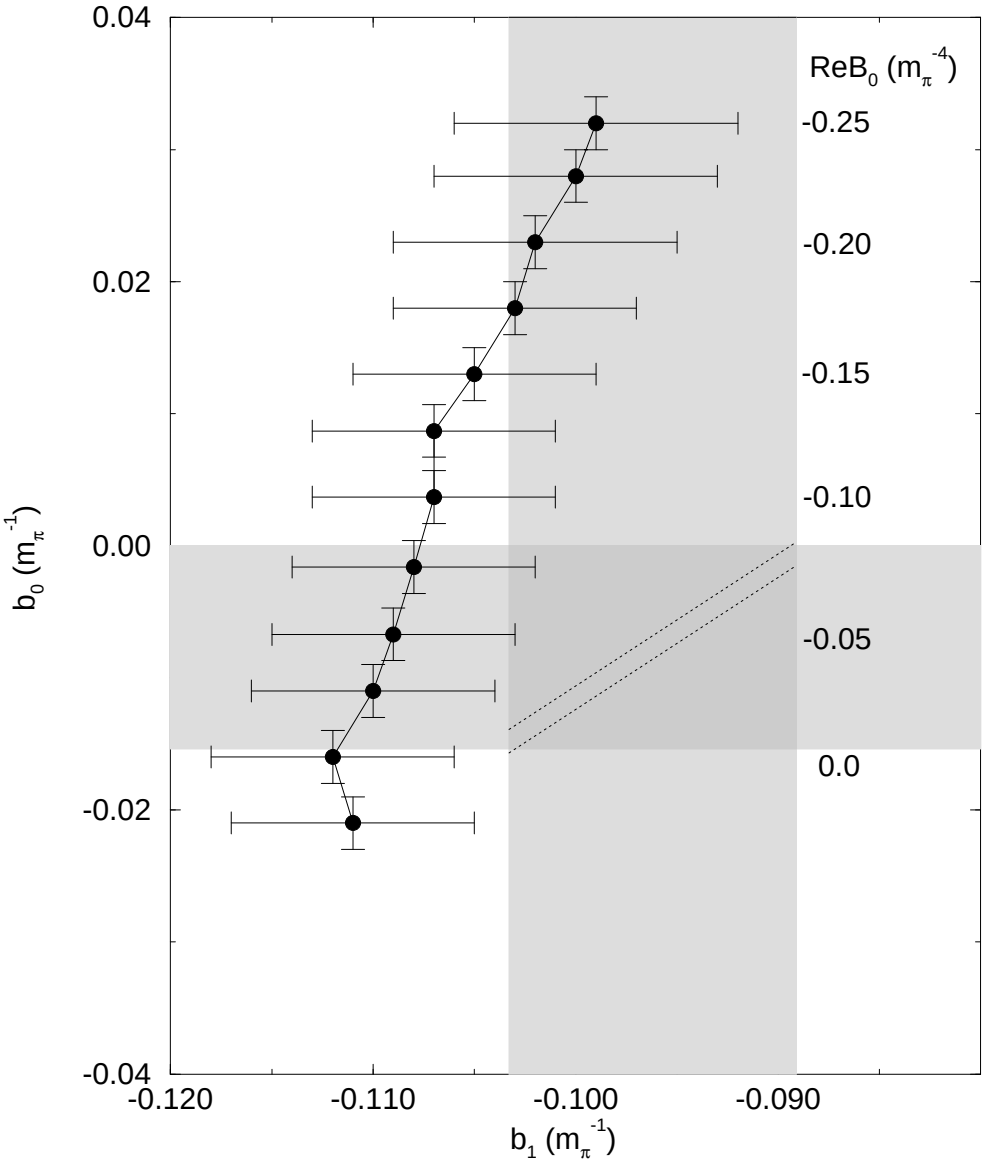

FIG. 1. Values of b_0 and b_1 obtained from χ^2 fits to normal pionic atom data are denoted by solid dots. Values of ReB_0 were held fixed during the fits and are listed in the figure (see text for details).

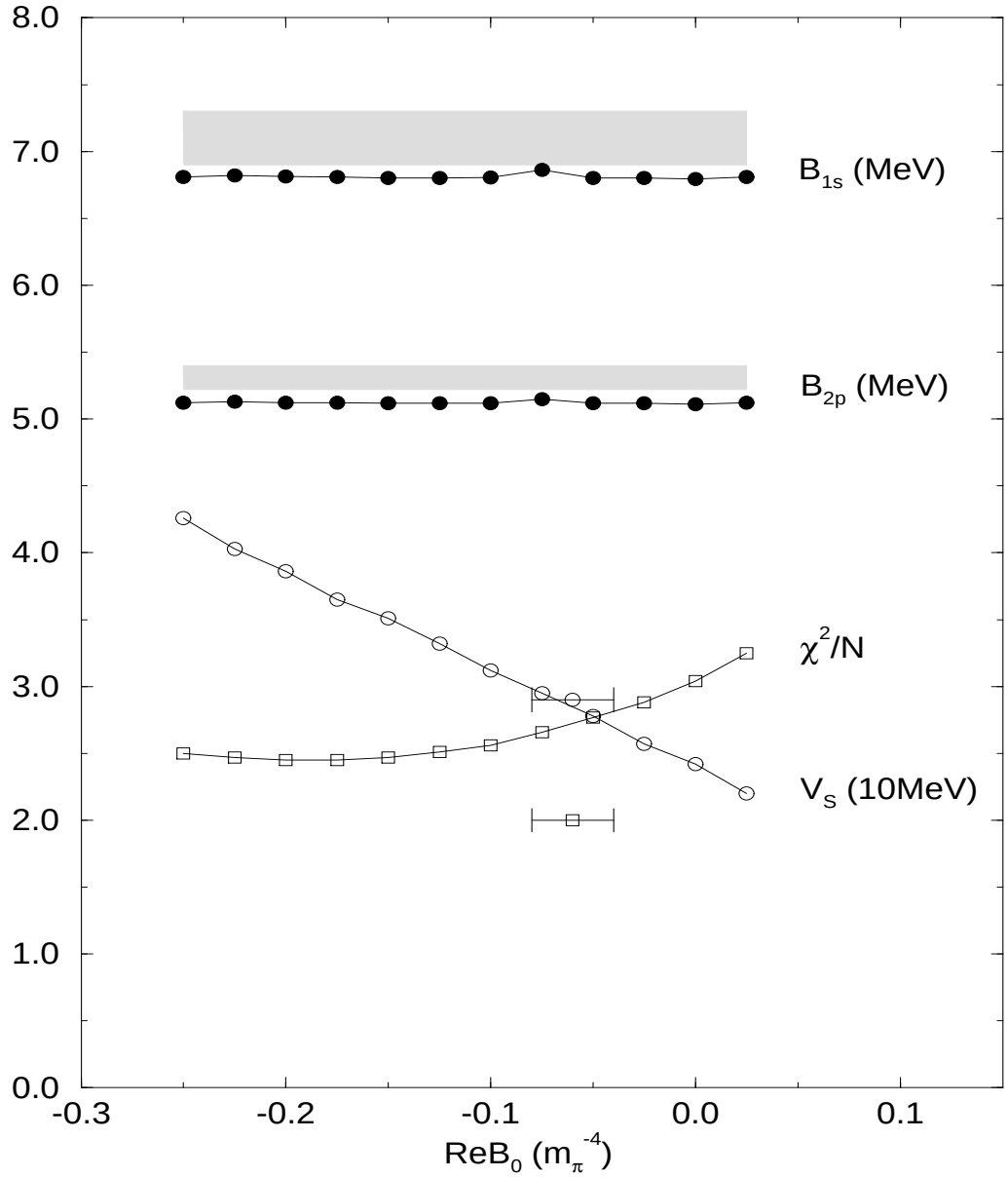

FIG. 2. Values of χ^2/N, of V_S and of the the calculated binding energies of pionic 1s and 2p states in ^{207}Pb as functions of ReB_0. The dots correspond to the points on Fig. 1. The shaded bands are the experimental results for the binding energies. The two horizontal error bars correspond to the unconstrained best fit.